High fidelity teleportation between light and atoms

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We show how high fidelity quantum teleportation of light to atoms can be achieved in the same setup as was used in the recent experiment [J. Sherson et.al., quant-ph/0605095, accepted by Nature], where such an inter-species quantum state transfer was demonstrated for the first time. Our improved protocol takes advantage of the rich multimode entangled structure of the state of atoms and scattered light and requires simple post-processing of homodyne detection signals and squeezed light in order to achieve fidelities up to 90% (85%) for teleportation of coherent (qubit) states under realistic experimental conditions. The remaining limitation is due to atomic decoherence and light losses.

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The first quantum teleportation of light on atoms was recently demonstrated by J. Sherson et.al. [1]. Based on a protocol proposed in [2], the experiment utilized entanglement between a cloud of atoms and a propagating pulse to transfer the coherent state carried by an other, independent pulse to the collective spin state of atoms. Measured fidelities ranging from 56% up to 64% clearly constitute a better-than-classical transfer of coherent states [3] and essentially prove that there indeed was entanglement of light and atoms present. If quantum teleportation of light on atoms was to be used as a building block of a quantum network, requirements on its performance will of course be more stringent. For example, recent security analyses of continuous variable quantum cryptography [4, 5] prove that the tolerable amount of excess noise will be in any case below 0.4 (0.8) shot noise units for protocols based on coherent (squeezed) states, corresponding to fidelities of 83% (71%). In this paper we elaborate on methods to improve the protocol of [1, 2] such as to attain high fidelity teleportation of light on atoms.

This goal can be achieved in two ways: First, a simple post-processing of homodyne detection signals recorded in the Bell measurement followed by a suitable feedback onto atoms already yields a significant enhancement. The strategy is based on the idea to include in the description also higher order temporal scattering modes, which were treated as noise in the original protocol [2]. In this way it is possible to benfit from the rich, multimode entangled structure inherent to the state of scattered light and atoms. Second, the remaining excess noise in atoms will be due to vacuum fluctuations of light, which can be reduced by using squeezed light already at the step of entangling light and atoms. Combined, these two methods will yield a fidelity which is limited by light losses and atomic decoherence only and can get close to 90% under realistic experimental conditions for teleportation of coherent states. We also study the teleportation of qubit

states, encoded in superpositions of vacuum and a single photon state, for which we predict a fidelity close to 85% under the same conditions. In the following we will first extend the model developed in [2] and give a complete description of the entangled state of light and atoms, which will be used as a resource in the teleportation protocols presented thereafter.

Resource state The system is the same as in [1, 2]. An ensemble of N_{at} Alkali atoms, whose ground state spins are maximally polarized along the x-direction, is immersed in a homogeneous magnetic field aligned along the same direction. The transverse collective spin components, Larmor precessing at a frequency Ω , can be described by canonical operators [X, P] = i with zero mean and a normalized variance $\Delta X^2 = \Delta P^2 = 1/2$ for the initial coherent spin state. A strong coherent pulse of frequency ω_0 and linearly polarized along x is then sent through the atomic sample along the z-direction. The scattered, ypolarized light is described in terms of spatially localized modes $[x(z), p(z')] = ic\delta(z-z')$. These modes are initially in vacuum such that $\langle x(z) \rangle = \langle p(z) \rangle = 0$ and $\langle x(z)x(z')\rangle = \langle p(z)p(z')\rangle = c\delta(z-z')/2$. The dynamics of this system can be described by the effective Hamiltonian [2, 6, 7]

$$H = H_{at} + H_{li} + V. (1)$$

 $H_{at} = \hbar\Omega(X^2 + P^2)/2$ describes the effect of the magnetic field, H_{li} the propagation of light and $V = \hbar\kappa Pp(0)/\sqrt{T}$ the effective interaction of light and atoms. T is the pulse length and κ a dimensionless coupling constant given by $\kappa = \sqrt{N_{ph}N_{at}F}a_1\sigma\Gamma/2A\Delta$ where N_{ph} is the overall number of photons in the pulse, a_1 is a constant characterizing the ground state's vector polarizability, σ is the scattering cross section, Γ the decay rate, A the effective beam cross section and Δ the detuning from the probed transition. Note that the effective form of the interaction V is true only in the case where Δ is larger than

the typical exited states' hyperfine splitting [6, 7]. In the following we will study the unitary evolution according to Hamiltonian (1). Atomic dephasing and light losses in this system are small and will be treated perturbatively as linear losses.

Hamiltonian (1) gives rise to a set of linear Maxwell-Bloch equations, whose explicit form can be found in [2]. These equations have to be integrated up to a time T, when the strong pulse triggering the interaction leaves the sample. This can be done along the lines of [2] and the solution can be phrased in terms of a set of temporal scattering modes defined as

$$x_{c,n}^{\rm in} = \frac{\mathcal{N}_n}{\sqrt{T}} \int_0^T d\tau \cos(\Omega \tau) \bar{P}_n(\tau/T) \bar{x}(c\tau, 0). \tag{2}$$

Here $n=0,1,2,\ldots,\bar{P}_n(x)=P_n(2x-1)$, where $P_n(x)$ is the n-th Legendre polynomial and \mathcal{N}_n is a normalization constant. Analogous definitions hold for $p_{c,n}^{\text{in}}$ with \bar{x} replaced by \bar{p} and for $x_{s,n}^{\text{in}}$, $p_{s,n}^{\text{in}}$ with $\cos(\Omega \tau)$ replaced by $\sin(\Omega \tau)$. In the limit of $\Omega T\gg 1$, which is well fulfilled under usual experimental conditions [1] where T is on the order of several ms and Ω of some 100 kHz, and for $n\ll\Omega T$ these modes are effectively orthogonal, $[x_{\alpha,n},p_{\beta,m}]=i\delta_{\alpha\beta}\delta_{n,m}$ $(\alpha,\beta=c,s)$, and the normalization is given by $\mathcal{N}_n=\sqrt{4n+2}$. By means of these modes the final state of atoms and scattered light can be expressed as

$$X^{\text{out}} = X^{\text{in}} + \frac{\kappa}{\sqrt{2}} p_{\text{c},0}^{\text{in}}$$
 (3a)

$$P^{\text{out}} = P^{\text{in}} + \frac{\kappa}{\sqrt{2}} p_{\text{s,0}}^{\text{in}} \tag{3b}$$

$$p_{\alpha,n}^{\text{out}} = p_{\alpha,n}^{\text{in}} \quad (n \ge 0, \, \alpha = c, s),$$
 (3c)

$$x_{\rm c,0}^{\rm out} = x_{\rm c,0}^{\rm in} + \frac{\kappa}{\sqrt{2}} P^{\rm in} + \left(\frac{\kappa}{2}\right)^2 \left(p_{\rm s,0}^{\rm in} - \frac{1}{\sqrt{3}} p_{\rm s,1}^{\rm in}\right), (3d)$$

$$x_{s,0}^{\text{out}} = x_{s,0}^{\text{in}} - \frac{\kappa}{\sqrt{2}} X^{\text{in}} - \left(\frac{\kappa}{2}\right)^2 \left(p_{c,0}^{\text{in}} - \frac{1}{\sqrt{3}} p_{c,1}^{\text{in}}\right), (3e^{-\frac{1}{2}} + \frac{1}{\sqrt{3}} p_{c,1}^{\text{in}})$$

$$x_{c,n}^{\text{out}} = x_{c,n}^{\text{in}} + \left(\frac{\kappa}{2}\right)^2 \left(\alpha_n p_{s,n-1}^{\text{in}} - \alpha_{n+1} p_{s,n+1}^{\text{in}}\right),$$
 (3f)

$$x_{s,n}^{\text{out}} = x_{s,n}^{\text{in}} - \left(\frac{\kappa}{2}\right)^2 \left(\alpha_n p_{c,n-1}^{\text{in}} - \alpha_{n+1} p_{c,n+1}^{\text{in}}\right),$$
 (3g)

where $\alpha_n = 1/\sqrt{4n^2-1}$ and the last two equations concern the cases $n \geq 1$ only. Obviously correlations between atoms and the zero order light modes are created proportional to κ . In addition, proportional to κ^2 , there are also correlations induced between the various higher order light modes. This back-action effect of light onto itself was clearly visible in the measurements of $x_{\alpha,0}$ performed in [1] and were well described by Eq. (3d) and (3e).

As opposed to [1, 2], where only the atomic mode and the n=0 light modes were considered as being part of the system, our aim here is to take advantage also of the correlations created among the higher order scattering modes in order to improve the teleportation fidelity. The protocol proceeds as follows:

Input state The quantum state to be teleported is encoded in a mode [y,q]=i given by

$$y = \sum_{n=0}^{N} \frac{c_n}{\sqrt{2}} (y_{s,n} + q_{c,n}), \quad q = \sum_{n=0}^{N} \frac{c_n}{\sqrt{2}} (q_{s,n} - y_{c,n}),$$
(4)

where the modes $[y_{\alpha,n}, q_{\beta,m}] = i\delta_{\alpha,\beta}\delta_{n,m}$ are defined analogously to Eq. (2) and the coefficients c_n are real and fulfill $\sum_n c_n^2 = 1$. This mode is centered in frequency at the upper sideband $\omega_0 + \Omega$ and has a slowly varying envelope determined by the coefficients c_n and N, which both will be specified later. The quantum state of this mode can in principle be arbitrary, but we will focus on coherent states in the following.

Bell measurement The scattered light in y-polarization, described by Eqs. (3d) to (3g), interferes at a balanced beam splitter with the input field. After the beam splitter the commuting observables

$$\tilde{x}_{c,n} = \frac{1}{\sqrt{2}} \left(x_{c,n}^{\text{out}} + y_{c,n} \right), \quad \tilde{x}_{s,n} = \frac{1}{\sqrt{2}} \left(x_{s,n}^{\text{out}} + y_{s,n} \right),$$

$$\tilde{q}_{c,n} = \frac{1}{\sqrt{2}} \left(p_{c,n}^{\text{out}} - q_{c,n} \right), \quad \tilde{q}_{s,n} = \frac{1}{\sqrt{2}} \left(p_{s,n}^{\text{out}} - q_{s,n} \right)$$
(5)

are measured up to n_{max} . This can be achieved by multiplying the photocurrent resulting from a standard polarimetric measurement of Stokes vector components S_y (or S_z) with the pulse envelopes given in Eq. (2) and integrating over the pulse duration.

Feedback Let the measurement outcomes corresponding to the observables above be given by $\tilde{X}_{c,n}$, $\tilde{X}_{s,n}$, $\tilde{Q}_{c,n}$ and $\tilde{Q}_{s,n}$ respectively. The atomic state is then displaced by an amount $\sum_n c_n(\tilde{X}_{s,n} - \tilde{Q}_{c,n})$ in X and $-\sum_n c_n(\tilde{X}_{c,n} + \tilde{Q}_{s,n})$ in P. In the ensemble average the final atomic state is then given by $X^{\text{fin}} = X^{\text{out}} + \sum_n c_n(\tilde{x}_{s,n} - \tilde{q}_{c,n})$ and $P^{\text{fin}} = P^{\text{out}} - \sum_n c_n(\tilde{x}_{c,n} + \tilde{q}_{s,n})$, such that, by means of Eqs. (3),(4) and (5) we arrive at

$$X^{\text{fin}} = y + \frac{1}{\sqrt{2}} \sum_{n=0}^{N} c_n x_{\text{s},n}^{\text{in}} + \left(1 - \frac{c_0 \kappa}{2}\right) X^{\text{in}} - \sum_{n=0}^{N+1} f_n p_{\text{c},n}^{\text{in}},$$

$$P^{\text{fin}} = q - \frac{1}{\sqrt{2}} \sum_{n=0}^{N} c_n x_{\text{c},n}^{\text{in}} + \left(1 - \frac{c_0 \kappa}{2}\right) P^{\text{in}} - \sum_{n=0}^{N+1} f_n p_{\text{s},n}^{\text{in}},$$
(6)

where the coefficients f_n are

$$f_0 = \frac{1}{\sqrt{2}} \left[c_0 - \kappa + \left(\frac{\kappa}{2} \right)^2 (c_0 + c_1 \alpha_1) \right],$$

$$f_n = \frac{1}{\sqrt{2}} \left[c_n + \left(\frac{\kappa}{2} \right)^2 (c_{n+1} \alpha_{n+1} - c_{n-1} \alpha_n) \right] \quad (n \ge 1)$$

and one has to set $c_{N+1} = c_{N+2} = 0$ in the sums in (6). As is evident from Eqs. (6), atoms receive the correct light mode (first terms on the r.h.s.) as well as a certain amount of excess noise (remaining three terms).

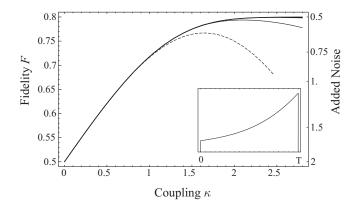


FIG. 1: Teleportation fidelity F and added noise in units of shot noise for coherent states versus coupling κ for different numbers of modes N included in the protocol, optimized for the parameters c_n determining the pulse shape of the input mode. The dashed line is the fidelity achievable if only the zeroth order mode, N=0, is included (c.f. [1, 2]). The thin line corresponds to N=1. The results for N=2 and N=3 (thick line) are practically identical and saturate the bound of F=80%. Inset: Optimal envelope for the input field for the case N=3, $\kappa=2$, normalized over pulse duration [0,T].

For unit-gain teleportation of coherent states, it is the variance of the latter terms which limits the teleportation fidelity (see [2] for a definition) and therefore has to be minimized - for a given coupling κ - by a proper choice of the coefficients c_n . Respecting the normalization condition $\sum_n c_n^2 = 1$ the best result that can be expected from such a strategy would be a cancelation of the last two terms in both of Eqs. (6). In this case the final atomic state would be $X^{\text{fin}} = y + \sum_{n} c_n x_{s,n}^{\text{in}} / \sqrt{2}$ and $P^{\text{fin}} = q - \sum_{n} c_n x_{c,n}^{\text{in}} / \sqrt{2}$, which amounts to half a unit of vacuum noise added to both spin components or a fidelity of 80%. Figure 1 shows the result of such an optimization for different choices of N, that is the number of modes which are included in the protocol. The limiting value of 80% can in deed be achieved by taking into account the first three higher order modes only. In order to beat also this limit, observe first that the half unit of vacuum noise added to the atomic state is due to the initial vacuum noise of modes $x_{\alpha,n}^{\text{in}}$, that is the vacuum field in y-polarization copropagating with the classical x-polarized pump field. These vacuum fluctuations can be suppressed by injecting squeezed light along with the classical field. The squeezing spectrum has to be broad enough such as to cover the sidebands at $\pm\Omega$ which is readily provided by a state of the art source of squeezed light, whose squeezing spectrum typically covers several MHz. Using squeezed light, the final atomic variance is $(\Delta X^{\text{fin}})^2 = (\Delta P^{\text{fin}})^2 = 1 + s/2$, where s is the squeezed variance of y-polarized light and the corresponding fidelity is F = 2/(2 + s/2), ranging from 80% for s=0 (no squeezing) approaching 100% for $s\to 0$. Figure 2 shows the result of protocols involving four tem-

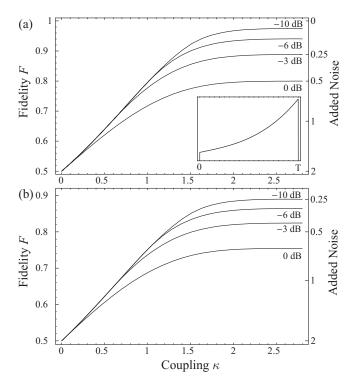


FIG. 2: (a) Teleportation fidelity F and added noise in units of shot noise for coherent states versus coupling κ for protocols involving four modes (N=3) using squeezing of light down to $-3\,\mathrm{dB}$, $-6\,\mathrm{dB}$ and $-10\,\mathrm{dB}$. The curve for $0\,\mathrm{dB}$ is identical to the thick line of Figure 1. Inset: Optimal envelope for the input field for $-10\,\mathrm{dB}$ of squeezing and $\kappa=2$, normalized over pulse duration [0,T].

(b) Teleportation fidelity F and added noise in units of shot noise for coherent states versus coupling κ including 10% atomic decay and 10% light losses. The optimal pulse shape is practically identical to the one shown in the inset of (a).

poral modes (N=3) and vacuum noise reduction down to s=0.5, s=0.25 and s=0.1 corresponding to about $-3\,\mathrm{dB}$, $-6\,\mathrm{dB}$ and $-10\,\mathrm{dB}$ of light squeezing respectively. Fidelities level off at the values expected from the simple formula given above and are thus bounded by the amount of single-mode squeezing, which reminds of the situation for continuous variable light-to-light teleportation [8, 9], whose performance is limited by the amount of two-mode squeezing.

Losses Up to this point we have neglected any effects of decoherence, which will inevitably occur due to spontaneous emission and absorption of light. As discussed in [2], these processes can be treated as linear losses, such that f.e. the state of atoms after the scattering is given

by

$$X^{\text{out}} = \sqrt{1 - \beta} \left(X^{\text{in}} + \frac{\kappa}{\sqrt{2}} p_{c,0}^{\text{in}} \right) + \sqrt{\beta} f_X,$$
$$P^{\text{out}} = \sqrt{1 - \beta} \left(P^{\text{in}} + \frac{\kappa}{\sqrt{2}} p_{s,0}^{\text{in}} \right) + \sqrt{\beta} f_P,$$

rather than by Eqs. (3a) and (3b). Passive light losses are naturally described by similar expressions for $x_{\alpha,n}^{\text{out}}$, $p_{\alpha,n}^{\text{out}}$. We assume that all light modes of interest are affected by the same amount of $(1-\epsilon)$ of power loss. Simple considerations show that in this case we can expect a fidelity of $F=2/(2+s/2+\beta+\epsilon)$. Figure 2 shows the numerical results including $\beta=10\%$ decay of transverse spin components and $\epsilon=10\%$ absorption losses in each light mode, which corresponds to the experimental conditions of [1]. The simple bound given above agrees again well with numerical results.

Non-unit gain teleportation So far we considered only unit-gain teleportation of coherent states, that is, we required that amplitudes are transmitted faithfully. If however it is known that the coherent states to be teleported are drawn from a certain pre-defined set only, such as a Gaussian distribution around the vacuum state, it might be advantageous to accept a certain mismatch in amplitude in order to reduce the added noise. The protocol described above is easily generalized to non-unit gain feedback by simply replacing the feedback coefficients c_n in Eqs. (6) by gc_n , where g is now a suitably chosen gain. The result will be

$$\begin{split} \langle X^{\,\mathrm{fin}} \rangle &= g \langle y \rangle \qquad (\Delta X^{\,\mathrm{fin}})^2 = g^2 \Delta y^2 + \Delta F^2, \\ \langle P^{\,\mathrm{fin}} \rangle &= g \langle q \rangle \qquad (\Delta P^{\,\mathrm{fin}})^2 = g^2 \Delta q^2 + \Delta F^2, \end{split} \tag{7}$$

where ΔF^2 represents the variance of the last three terms in Eqs. (6). From this one can evaluate the teleportation fidelity, averaged over the set of input states, and optimize for g. As an example, for a Gaussian distributed set of input states of mean photon number $\bar{n}=2$, the average fidelity can be 90% for $\kappa=1.5$ and light squeezing of 10 dB including 10% atomic dephasing and 10% light losses. The optimal gain in this case is g=0.9.

Teleportation of qubit states Having discussed the teleportation of coherent states, it is interesting to ask how well the proposed protocol would work, if the state to be teleported was a non-gaussian state, f.e. a qubitstate $|\psi(\theta,\phi)\rangle = \cos(\theta/2)|0\rangle + \exp(i\phi)\sin(\theta/2)|1\rangle$, where $|0\rangle$ and $|1\rangle$ are the vacuum and the single-photon state of mode (4) respectively. It is clear that unit fidelity of coherent state teleportation implies unit fidelity for the teleportation of arbitrary states, as coherent states provide a basis in Hilbert space. But also for imperfect protocols one can expect a rather high teleportation fidelity for states involving only few photons given the result of the previous paragraph. In order to explicitly evaluate the fidelity for states of the form $|\psi(\theta,\phi)\rangle$, note that the whole teleportation protocol implements a completely positive map \mathcal{E} such that the (density operator

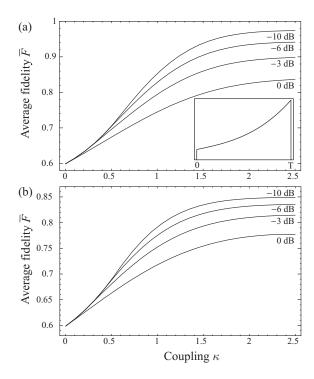


FIG. 3: (a) Teleportation fidelity F for qubit states versus coupling κ for protocols involving four modes (N=3) using squeezing of light down to $-3\,\mathrm{dB}, -6\,\mathrm{dB}$ and $-10\,\mathrm{dB},$ averaged over the Bloch sphere. Inset: Optimal envelope for the input field for $-10\,\mathrm{dB}$ of squeezing and $\kappa=2$, normalized over pulse duration [0,T].

(b) Teleportation fidelity F for qubit states versus coupling κ including 10% atomic decay and 10% light losses, averaged over the Bloch sphere. The optimal pulse shape is practically identical to the one in shown in the inset of (a).

of the) state of atoms after the teleportation protocol is $\mathcal{E}(|\psi\rangle\langle\psi|)$ when $|\psi\rangle$ was the input. The map \mathcal{E} is a general Bogoliubov transformation and fixed by the linear input-output relations (6). Evaluating the corresponding fidelity, $F = \langle \psi | \mathcal{E}(|\psi\rangle\langle\psi|) | \psi\rangle$, is not straight forward, as \mathcal{E} - loosely speaking - mixes creation and annihilation operators. In order to calculate the fidelity F one can take advantage of (i) relation

$$|n\rangle\langle m| = \left[\frac{\partial^{n+m}}{\partial\alpha^n\partial\alpha^{*m}}\left(e^{\alpha\alpha^*}|\alpha\rangle\langle\alpha|\right)\right]\bigg|_{\alpha=\alpha^*=0}, (n,m=0,1),$$

where $|n\rangle$ is a Fock state and $|\alpha\rangle$ a coherent state and (ii) the fact that the action of \mathcal{E} on coherent states has the simple representation

$$\mathcal{E}(|\alpha\rangle\langle\alpha|) = \frac{1}{2\pi\sigma^2} \int d^2\beta e^{-|\beta-g\alpha|^2/2\sigma^2} |\beta\rangle\langle\beta|,$$

where σ^2 is given by the variance of atomic spin components c.f. Eqs. (7), that is $\sigma^2 = [(\Delta X^{\rm fin})^2 - 1]/4 = [(\Delta P^{\rm fin})^2 - 1]/4$ with variances measured in units of vacuum noise. By means of these relations the fidelity, av-

eraged over the Bloch sphere, is found to be

$$\bar{F} = \frac{1}{4\pi} \int d\Omega \langle \psi | \mathcal{E}(|\psi\rangle \langle \psi|) | \psi \rangle$$

$$= \frac{3 + 2g + g^2 + 2(9 + 2g - 3g^2)\sigma^2 + 24\sigma^4}{6(1 + 2\sigma^2)^3}.$$
(8)

This expression can now again be optimized with respect to the gain g and the input-envelope fixed by c_n . The results are shown in Figure 3 and prove that it is well possible to violate the classical benchmark of 2/3 for the teleportation of qubit states. Note that relation (8) holds for all Gaussian maps of the form (7) and is thus of relevance also in other situations such as for example for evaluating the efficiency of quantum memory protocols [10, 11, 12, 13].

In summary we showed that the protocol used in [1] to perform teleportation of light on atoms can be improved to yield high fidelities up to 90% under realistic conditions. The final limitation comes from decoherence of atoms and light losses which both are on the 10% level in the setup of [1], which is a room-temperature ensemble of atoms in a glass cell. In particular in the balance of light losses 5% are due to propagation losses and detector inefficiencies and 5% come from reflections from walls of the glass cell. Losses of the latter kind can be reduced down to 0.5% with improved anti-reflection coating. Furthermore, we expect that for cold trapped atoms, eventually in an optical lattice, both atomic dephasing and light losses can be diminished significantly. For a different proposal of light-to-atom teleportation based on collective recoil in a Bose-Einstein condensate see [14]

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